

AP Calculus AB

Implicit Differentiation (Day 2)

1) $x^2y + xy^2 = 6$

TWO SEPARATE
PRODUCT RULES

$$x^2 \frac{dy}{dx} + 2xy + x(2y \frac{dy}{dx}) + y^2 = 0$$

$$x^2 \frac{dy}{dx} + 2xy \frac{dy}{dx} = -y^2 - 2xy$$

$$\frac{dy}{dx} = \frac{-y^2 - 2xy}{x^2 + 2xy}$$

2) $y^2 = \frac{x-1}{x+1}$

$$2y \frac{dy}{dx} = \frac{(x+1) - (x-1)}{(x+1)^2}$$

$$2y \frac{dy}{dx} = \frac{2}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{1}{y(x+1)^2}$$

3) $x = \tan y$

$$1 = \sec^2 y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \cos^2 y$$

4) $x + \sin y = xy$

$$1 + \cos y \cdot \frac{dy}{dx} = x \frac{dy}{dx} + y$$

$$\cos y \frac{dy}{dx} - x \frac{dy}{dx} = y - 1$$

$$\frac{dy}{dx} = \frac{y-1}{\cos y - x}$$

5) $x^2 - xy = 5$

$$2x - x \frac{dy}{dx} - y = 0$$

$$-x \frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx} = \frac{2x-y}{x}$$

6) $y = x^{9/4}$

$$\frac{dy}{dx} = \frac{9}{4} x^{5/4}$$

7) $y = \sqrt[3]{x}$

$$\frac{dy}{dx} = \frac{1}{3} x^{-2/3}$$

8) $y = (2x+5)^{-1/2}$

$$\frac{dy}{dx} = -\frac{1}{2} (2x+5)^{-3/2} \cdot 2$$

$$9) x^3 + y^3 = 18xy$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 18x \frac{dy}{dx} + 18y$$

$$3y^2 \frac{dy}{dx} - 18x \frac{dy}{dx} = 18y - 3x^2$$

$$\frac{dy}{dx} (3y^2 - 18x) = 18y - 3x^2$$

$$\frac{dy}{dx} = \frac{18y - 3x^2}{3y^2 - 18x}$$

$$\frac{dy}{dx} = \frac{6y - x^2}{y^2 - 6x}$$

$$10) x^2 + y^2 = 13 \quad (-2, 3)$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\left. \frac{dy}{dx} \right|_{(-2, 3)} = \frac{2}{3}$$

$$12) x^2 - y^2 = 9$$

$$2x - 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y - x \frac{dy}{dx}}{y^2}$$

$$= \frac{y - x \left(\frac{x}{y} \right)}{y^2}$$

$$11) x^2 + xy - y^2 = 1$$

$$2x + x \frac{dy}{dx} + y - 2y \frac{dy}{dx} = 0$$

point

$$4 + 2y - y^2 = 1$$

$$-4 - 2y + y^2 = -1$$

$$y^2 - 2y - 3 = 0$$

$$(y-3)(y+1) = 0$$

$$y = 3 \quad y = -1$$

$$(2, 3) \quad (2, -1)$$

slope

$$x \frac{dy}{dx} - 2y \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x - 2y}$$

(2, 3)

$$\frac{dy}{dx} = \frac{-7}{-4} = \frac{7}{4}$$

(2, -1)

$$\frac{dy}{dx} = \frac{-3}{4}$$

$$y - 3 = \frac{7}{4}(x - 2)$$

$$y + 1 = -\frac{3}{4}(x - 2)$$

$$13) y' = x^2 + 4x - 2$$

Answers will vary.